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The Origin of a Peculiar Extra $U(1)$

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Abstract

The origin of a family-independent "extra $U(1)$", discovered by Barr, Bednarz, and Benesh and independently by Ma, and whose phenomenology has recently been studied by Ma and Roy, is discussed. Even though it satisfies anomaly constraints in a highly economical way, with just a single extra triplet of leptons per family, this extra $U(1)$ cannot come from four-dimensional grand unification. However, it is shown here that it can come from a Pati-Salam scheme with an extra $U(1)$, which explains the otherwise surprising cancellation of anomalies.

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The possibility of "extra U(1)" gauge groups has been much discussed. By extra $U(1)$ is meant a $U(1)$ factor in addition to the Standard Model group $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$. If the extra $U(1)$ (often called $U(1)'$) is broken near the weak scale, then the "extra Z" gauge boson associated with it (often called Z') can lead to interesting phenomenology [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13].

Grand unified theories (GUTs) based on groups with rank greater than 4 can lead to such $U(1)$ factors. For instance, $SO(10)$ contains an extra $U(1)$ that could survive to low energies (compared to the GUT scale), and E_6 contains two extra $U(1)$ symmetries, both of which, or some linear combination of which, could survive. These particular $U(1)'$ groups and their associated Z' bosons have been especially well studied, and in fact are the standard examples of extra $U(1)$ groups. In this paper we discuss a peculiar $U(1)'$ that is quite simple and economical, but does not come from grand unification. The existence of this $U(1)'$ was first discovered in a little-known paper by one of the present authors [14], (see Eq. (30) and the conclusions of that paper). Recently, it has been rediscovered by E. Ma [15], and its phenomenology studied by E. Ma and D.P. Roy [16]. Here we explain how this $U(1)'$ can arise in a simple Pati-Salam scheme of quark-lepton unification, which explains group-theoretically speaking, why this surprising anomaly-free $U(1)'$ exists. We also discuss how it may be related to grand unified models in higher space-time dimensions.

The existence of a $U(1)'$ brings with it additional anomaly cancellation conditions. (These are given a very general analysis in [14].) If the group is $G_{SM} \times U(1)'$ there are six new anomaly conditions, which in an obvious notation are the $(3_c)^2 1'$, $(2_L)^2 1'$, $(1_Y)^2 1'$, $1_Y (1')^2$, $(1')^3$, and $(gravity) 1'$ conditions. (For a general analysis of solutions to these conditions, see [14].) If $U(1)'$ is family-independent and there are N multiplets of G_{SM} per family, then there are N quantities (i.e. the $U(1)'$ charges of these multiplets) that have to satisfy six conditions. At first glance, it would seem that there would be solutions if $N \geq 6$. However, in fact N must be at least 8, generally speaking, for a solution to exist. The reason is the following. First, the anomaly conditions are homogeneous and therefore do not fix the overall normalization of the $U(1)'$ charges of the fermion multiplets. Second, given any non-trivial solution for the $U(1)'$ charges, say $Q'(f) = X_f$ (where Q' is the generator of $U(1)'$ and f is any fermion), then $Q'(f) = X_f + \alpha Y(f)$ must also be a solution, where Y is the hypercharge and α is a free parameter. (Note that the $U(1)'$ charges given for the extra-lepton-triplet $U(1)$ in [14] are in fact a linear combination of the charges given here with hypercharge.)

Since two continuous parameters do not get fixed, there must be 8 rather than six charges, generally speaking, for a non-trivial solution to the anomaly conditions to exist, and thus 8 multiplets per family. (No matter how many multiplets there are, there is always the trivial solution $Q'(f) = \alpha Y(f)$ [14, 17, 18].)

Let us now ask what happens if there is only *one* extra fermion multiplet per family in addition to the usual five (namely $(u,d)_L$, u^c_L, d^c_L, $(\nu, e^-)_L$, and e^+_L). From the foregoing, it follows that there should not be any non-trivial solutions for a $U(1)'$ except, perhaps, in special cases. In fact, there are exactly two special cases. One is the well-known extra $U(1)$ that comes from $SO(10)$. The other is the peculiar $U(1)$ that we are discussing in this paper.

If there is only one extra fermion multiplet per family, then (assuming the gauge quantum numbers of all the families to be the same) the extra multiplet must be a in a real representation of G_{SM}. Call it $(R_3, R_2, 0)$. Define the ratios $r_3 \equiv C_3(R_3)/R_3$ and $r_2 \equiv C_2(R_2)/R_2$ (where the Casimirs are normalized so that $C_3(3) = C_2(2) = \frac{1}{2}$). It can be shown that there is no solution to the complete set of anomaly equations unless two non-trivial conditions are satisfied by the dimensions and Casimirs of the extra multiplet:

$$0 = -\frac{44}{9} r_3^2 + 2 r_3 r_2 + \frac{3}{2} r_2^2 + \frac{4}{3} r_3 - r_2, \tag{1}$$

and

$$0 = -\frac{32}{3} r_3^3 + \frac{61}{9} r_3^2 r_2 + \frac{13}{2} r_3 r_2^2 + r_2^3 + 4 r_3^2 - \frac{17}{3} r_3 r_2 - 2 r_2^2 + r_2 + \frac{1}{12} \left(\frac{1}{R_3^2 R_2^2} - 1 \right). \tag{2}$$

(These equations can be derived by solving the four linear anomaly conditions to express all the $U(1)'$ charges in terms of two parameters, and then substituting the result into the quadratic and cubic anomaly conditions. When this is done, the unfixed parameters drop out, as they should, and Eqs. (1) and (2) result.) That two non-trivial conditions have to be satisfied by the choice of representation just follows from the fact that there are only 6 multiplets rather than the 8 per family that is needed in the general case for a solution.

If the extra multiplet is a color singlet, then $R_3 = 1$, $r_3 = 0$, and the equations reduce to $(\frac{3}{2} r_2 - 1) r_2 = 0$ and $r_2^3 - 2 r_2^2 + r_2 + \frac{1}{12}(R_2^{-2} - 1) = 0$. These have only two solutions: either $R_2 = 1$ ($\Rightarrow r_2 = 0$), or $R_2 = 3$ ($\Rightarrow r_2 = \frac{2}{3}$). There are no solutions if R_3 is a non-singlet. Thus, the only two solutions are the following [15]:

Solution 1: If the extra fermion multiplet in each family is just a $(1, 1, 0)$ of G_{SM} (i.e. a "right-handed neutrino"), then there is the solution $Q'((u, d)_L,\ u^c_L, d^c_L, (\nu, e^-)_L, e^+_L, N^c_L) = (+1, +1, -3, -3, +1, +5)$. This solution can be understood group-theoretically: the Q' is just the generator of the extra $U(1)$ contained in $SO(10)$.

Solution 2: If the extra multiplet in each family is a $(1, 3, 0)$ of G_{SM}, i.e. a lepton triplet, which we shall denote (t^+, t^0, t^-), then there is the following simple solution which we will call the "extra-lepton-triplet $U(1)$" or $U(1)_{ELT}$:

$$
\begin{array}{llll}
Q' = +1 : & \begin{pmatrix} u \\ d \end{pmatrix}_L , & \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L , & e^+_L, \\
 & & & \\
Q' = -1 : & u^c_L, & d^c_L, & \begin{pmatrix} t^+ \\ t^0 \\ t^- \end{pmatrix}_L .
\end{array}
\tag{3}
$$

The $(3_c)^2 1'$ anomaly condition is obviously satisfied, since the contributions from the u and d cancel those from the u^c and d^c. The $(1')^3$ and $(gravity)1'$ conditions are both satisfied because there happen to be nine (= 6+2+1) fields with $Q' = +1$ and nine ($= 3 + 3 + 3$) fields with $Q' = -1$. The $(2_L)^2 1'$ condition is satisfied because the contribution of the $SU(2)_L$ triplet $(t^+, t^0, t^-)_L$ cancels the contribution of the four (counting color) $SU(2)_L$ doublets $(u, d)_L$ and $(\nu, e^-)_L$ (since $C_2(3) = 4C_2(2)$). Finally, the $(1_Y)^2 1'$ condition is satisfied because $6(\frac{1}{6})^2 + 2(-\frac{1}{2})^2 + 1(+1)^2 - 3(-\frac{2}{3})^2 - 3(\frac{1}{3})^2 - 3(0)^2 = \frac{5}{3} - \frac{5}{3} = 0$.

As one can see, these anomaly cancellations are non-trivial, and it seems rather remarkable that such a solution exists, since this set of fermions and charges is *not* embeddable in a grand unified theory, unlike the previous case. (It is not difficult to see why it is not embeddable in a grand unified theory. An $SU(2)_L$ triplet is in the symmetric product of two doublets, and thus can come from a multiplet that is in a symmetric product of two **5**s of $SU(5)$. However, such an $SU(5)$ multiplet will also contain a **6** of color.) It thus appears at first glance that the existence of this solution to the anomaly conditions is merely a fluke. However, as we shall now see, this solution can be related to a Pati-Salam model [19] with an extra $U(1)$ in a way that makes it seem a little less surprising.

Suppose we consider a model with the group $G_{PS} \times U(1)' = SU(4)_c \times SU(2)_L \times SU(2)_R \times$

$U(1)'$ and fermion multiplets

$$
\begin{aligned}
Q' = +1: &\begin{pmatrix} u & \nu \\ d & e^- \end{pmatrix}_L = (4,2,1)^{+1}_L, \quad \begin{pmatrix} e^+ \\ T^0 \\ T^- \end{pmatrix}_L = (1,1,3)^{+1}_L, \\
Q' = -1: &\begin{pmatrix} u^c & T^{c0} \\ d^c & T^+ \end{pmatrix}_L = (\overline{4},1,2)^{-1}_L, \quad \begin{pmatrix} t^+ \\ t^0 \\ t^- \end{pmatrix}_L = (1,3,1)^{-1}_L.
\end{aligned} \tag{4}
$$

Note that the Pati-Salam multiplets $(4,2,1)+(\overline{4},1,2)$ *cannot* come from a spinor of $SO(10)$ because they have opposite $U(1)'$ charges. Thus this model cannot come from a grand unified model (though, as we shall see, it can be related to one in higher dimensions). The $(4_c)^2 1'$, $(1')^3$, and $(gravity)1'$ anomalies cancel in an obvious way. The $(2_L)^2 1'$ and $(2_R)^2 1'$ anomalies cancel because (as noted above) one $SU(2)$ triplet contributes with the same weight as four doublets. There are in this model five anomaly conditions that involve $U(1)'$, and they are satisfied by only three charge ratios. Nevertheless, the anomalies cancellation here is more transparent than in Eq. (3).

Now imagine that a Higgs field Ω in a $(4,1,2)^0$ representation of $G_{PS} \times U(1)'$ has a Yukawa coupling to the fermion multiplets $(\overline{4},1,2)^{-1}_L$ and $(1,1,3)^{+1}_L$. If its electrically neutral component gets a vacuum expectation value of magnitude M_R, then the pairs $(T^{c0}T^0)$ and (T^+T^-) will obtain mass of order M_R, and the group $G_{PS} \times U(1)'$ will break to $G_{SM} \times U(1)'$ with exactly the residual light fermion content shown in Eq. (3).

In the $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)'$ model, lepton masses arise from the following matrices:

$$
\begin{aligned}
\mathcal{L}_{lepton} &= (e_i^+, t_i^+) \begin{pmatrix} \gamma_{ij}\langle H_2^{(-2)}\rangle & \delta_{ij}\langle H_3^{(0)}\rangle \\ \sqrt{2}\beta_{ij}\langle H_2^{(0)}\rangle & \alpha_{ij}\langle H_1^{(2)}\rangle \end{pmatrix} \begin{pmatrix} e_j^- \\ t_j^- \end{pmatrix} \\
&+ \tfrac{1}{2}(\nu, t^0) \begin{pmatrix} \kappa_{ij}\langle H_3^{(-2)}\rangle & \beta_{ij}^T\langle H_2^{(0)}\rangle \\ \beta_{ij}\langle H_2^{(0)}\rangle & \alpha_{ij}\langle H_1^{(2)}\rangle \end{pmatrix} \begin{pmatrix} \nu \\ t^0 \end{pmatrix}.
\end{aligned} \tag{5}
$$

The i,j are family indices. The notation $H_R^{(m)}$ refers to a Higgs field in an R-component multiplet of $SU(2)_L$ (i.e. one with weak isospin $\frac{R-1}{2}$) that has $U(1)'$ charge $Q' = m$. The ρ

parameter tells us that the VEVs of $SU(2)_L$ triplet Higgs, if there are any, must be small compared to the weak scale. Let us therefore neglect the entries proportional to $H_3^{(0)}$ and $H_3^{(-2)}$ in Eq. (5). The singlet VEV $\langle H_1^{(2)} \rangle$ must, on the other hand, be large compared to the weak scale, since it generates the masses of the triplet leptons and the mass of the Z'. How large depends on the entries in Eq. (5) proportional to $\langle H_2^{(0)} \rangle$. There are two interesting cases.

Case 1. If the entries proportional to $\langle H_2^{(0)} \rangle$ in Eq. (5) are of order the weak scale, then it must be that $\langle H_1^{(2)} \rangle$ is superlarge (of order 10^{14} GeV or more) in order to keep the observed neutrino masses small. This gives the usual neutrino seesaw mechanism, except that the superheavy "right-handed neutrinos" are part of triplets of superheavy leptons. In this case, since $U(1)'$ is broken at superlarge scales, there is no interesting Z' phenomenology, and the mixing of the known leptons with the triplet leptons is negligible. There could, however, be interesting consequences for leptogenesis.

Case 2. If the entries proportional to $\langle H_2^{(0)} \rangle$ in Eq. (5) vanish (or are extremely small compared to the weak scale), then $\langle H_1^{(2)} \rangle$ can have any value from the Planck scale down to about a TeV. If the scale of $U(1)'$ breaking is in the TeV range, there would be interesting Z' phenomenology, which has been analyzed very thoroughly in [15], [16]. It should be observed that a $U(1)'$-neutral Higgs doublet $H_2^{(0)}$ is needed to give mass to the up quarks and down quarks, as can be seen from Eq. (1). Consequently, if the entries in Eq. (5) that are proportional to $\langle H_2^{(0)} \rangle$ are to vanish or be small *naturally*, some symmetry reason for this suppression must exist. One such reason could be a Z_2 symmetry under which the triplet leptons are odd and all other quarks and leptons (and all Higgs fields) are even. Note that this would prevent the triplet leptons from mixing with the ordinary leptons, and render them stable. They might then be candidates for dark matter. It is also possible, however, to construct models in which there is significant mixing of the triplet leptons and ordinary doublet and singlet leptons, for example through small entries proportional to $\langle H_3^{(0)} \rangle$ or through higher dimension operators. This mixing could lead to observable violations of the universality of the weak interactions of the leptons.

Obviously the ideas mentioned here (some of which were discussed in great detail in [15], [16]) do not exhaust all the possibilities, but they illustrate some of the ways in which lepton phenomenology could be different if there is the extra lepton triplet $U(1)$.

Another way in which these lepton triplets could be significant is in their effect on the running of the gauge couplings. In fact, they could allow unification of the Standard Model gauge couplings without low-energy supersymmetry. This may seem inconsistent with the fact, noted earlier, that the $U(1)_{ELT}$ cannot come from grand unification. However, it is consistent with non-supersymmetric unification of the standard model gauge groups in extra dimensions [20, 21]. For example, suppose a fifth dimension is compactified on a S_1/Z_2 orbifold, in such a way that an $SO(10) \times U(1)'$ gauge symmetry in the bulk is broken by the compactification to the $G_{PS} \times U(1)'$ subgroup. This happens if the gauge bosons of the coset $SO(10)/G_{PS}$ are odd under the Z_2. On the branes that bound the bulk, since they are fixed points of the Z_2, there is only the gauge symmetry $G_{PS} \times U(1)'$. Thus, the quarks and leptons living on one of these branes can have any $G_{PS} \times U(1)'$ quantum numbers consistent with anomaly cancellation. In particular, they could have the quantum numbers shown in Eq. (4). The Standard Model gauge couplings (to the extent that one can neglect corrections from gauge kinetic terms on the branes) would be unified due the $SO(10)$ symmetry in the bulk. It should be noted that even if the gauge unification happened at a scale much below 10^{15} GeV proton decay would not be a problem, since the gauge-mediated proton decay comes from gauge bosons in the coset $SO(10)/G_{PS}$, which do not live on the branes where the quarks and leptons exist

In such a brane scenario, it is not necessary that all three families have the structure shown in Eq. (4). It could be, for instance, that some of the families contain lepton triplets and the $U(1)'$ charges of Eq. (4), while the remaining families have no lepton triplets and are neutral under $U(1)'$. These latter families could exist either on the branes or in the bulk. However, if they existed in the bulk or mixed with fermions in the bulk, one would have to worry about the proton decay rate.

Assuming unification of gauge couplings fixes the mass of the triplet leptons (assuming all the triplet leptons have nearly the same mass). The mass of the triplets depends, of course, on how many of the families have triplets. In numerical examples presented in Table I we consider running of the gauge couplings at the one-loop level between the electroweak scale and grandunifying scale with n_3 extra triplets. We impose exact unification and use central values for $\sin^2\theta_w$, α_{em}^{-1} and α_s as given in [22] to determine intermediate scale where triplets reside. Clearly, similar analysis can also be performed in the orbifold scenario if the full particle content of the bulk and brane(s) is specified.

TABLE I: Triplet mass versus number of triplets (n_3) in case of one ($n = 1$) and two ($n = 2$) light Higgs doublets. Grandunifying scale is 2.5×10^{14} GeV and 1.9×10^{14} GeV respectively.

	$n = 1$	$n = 2$
$n_3 = 1$	7.2×10^6 GeV	9.3×10^7 GeV
$n_3 = 2$	4.2×10^{10} GeV	1.3×10^{11} GeV
$n_3 = 3$	7.6×10^{11} GeV	1.5×10^{12} GeV

Of course, it is possible that there could be extra lepton triplets at low or intermediate energy without there being any extra $U(1)'$ gauge group. However, if the only symmetry is G_{SM}, then there is nothing to "protect" the extra triplets from acquiring superlarge mass, since they are in a real representation of G_{SM}. The extra lepton triplet $U(1)$ does protect these triplets from acquiring such masses. Of course, in a non-supersymmetric model, the mass of the Higgs doublet is fine-tuned anyway; so one could argue that nothing is lost by fine-tuning the masses of the lepton triplets to be small also. However, the Higgs mass fine-tuning could be explained, in principle, "anthropically", whereas it is less obvious that the low masses of lepton triplets could be explained so easily in that way.

In conclusion, we have shown that the remarkable extra $U(1)$ that was discovered in [14] and [15] and analyzed in [16] has a simple group-theoretical explanation in terms of Pati-Salam unification, and may even be related to grand unification in higher dimensions. If it is related to grand unification in higher dimensions, then the scale at which the $U(1)'$ is broken is too large for the Z' phenomenology to be directly accessible in the near future. However, the extra heavy leptons would still be relevant to neutrino physics and possibly to leptogenesis. If the scheme comes from Pati-Salam unification, but not grand unification, then the scale of the $U(1)'$ breaking can have any value, and can, in particular, be near the TeV scale.

[1] N. G. Deshpande and D. Iskandar, Phys. Rev. Lett. **42**, 20 (1979).

[2] R. W. Robinett and J. L. Rosner, Phys. Rev. D **25**, 3036 (1982) [Erratum-ibid. D **27**, 679 (1983)].

[3] C. N. Leung and J. L. Rosner, Phys. Rev. D **29**, 2132 (1984).

[4] L. S. Durkin and P. Langacker, Phys. Lett. B **166**, 436 (1986).

[5] V. D. Barger, N. G. Deshpande and K. Whisnant, Phys. Rev. Lett. **56**, 30 (1986).

[6] D. London and J. L. Rosner, Phys. Rev. D **34**, 1530 (1986).

[7] U. Amaldi *et al.*, Phys. Rev. D **36**, 1385 (1987).

[8] G. Costa, J. R. Ellis, G. L. Fogli, D. V. Nanopoulos and F. Zwirner, Nucl. Phys. B **297**, 244 (1988).

[9] J. L. Hewett and T. G. Rizzo, Phys. Rept. **183**, 193 (1989).

[10] K. T. Mahanthappa and P. K. Mohapatra, Phys. Rev. D **43**, 3093 (1991) [Erratum-ibid. D **44**, 1616 (1991)].

[11] P. Langacker and M. x. Luo, Phys. Rev. D **45**, 278 (1992).

[12] T. Appelquist, B. A. Dobrescu, and A. R. Hopper, Phys. Rev. D **68**, 035012 (2003).

[13] M. Carena, A. Daleo, B. A. Dobrescu, and T. M. P. Tait, Phys. Rev. D **70**, 093009 (2004).

[14] S. M. Barr, B. Bednarz and C. Benesh, Phys. Rev. D **34**, 235 (1986).

[15] E. Ma, Mod. Phys. Lett. A **17**, 535 (2002).

[16] E. Ma and D. P. Roy, Nucl. Phys. B **644**, 290 (2002).

[17] K. T. Mahanthappa and P. K. Mohapatra, Phys. Rev. D **42**, 2400 (1990).

[18] V. A. Kostelecky and S. Samuel, Phys. Lett. B **270**, 21 (1991).

[19] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974).

[20] Y. Kawamura, Prog. Theor. Phys. **103**, 613 (2000).

[21] Y. Kawamura, Prog. Theor. Phys. **105**, 691 (2001).

[22] S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004).